CALCULATION OF NEUTRON SEPARATION ENERGY OF $^{12}_{6}C$

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Abstract

Nuclear structure calculations of ^{12}C has been performed by assuming the ^{12}C as a two-body system which is composed of a neutron and a core nucleus ^{11}C. The interaction between the neutron and core nucleus is derived by folding the nucleon-nucleon potential of a Gaussian form with the density distribution of the core nucleus. It is found that the neutron separation energy is in good agreement with the experimental value.

Keywords: neutron separation energy

Introduction

One of most fundamental aspects in nuclear physics is to get knowledge about nucleon-nucleon interactions. Sources which provide information about nucleon-nucleon interactions are scattering data and nuclear binding energies (Willians W.S.C.). In this work, we have determined the last neutron separation energy of ^{12}C by assuming that a nucleon moves in a nuclear potential which is an average effect of the other nucleons.

Since every nucleon is in bound state, the potential is expected to be a potential well. Each nucleon moves in an orbit which is the orbit of a single particle in that potential well. The mean free path of an energetic nucleon moving in nuclear matter is about 2 fm.

The Pauli principle strongly suppressed collisions between nucleons and that provides nearly undisturbed orbits for the nucleons in a nuclear matter. In nuclei the mean field is exclusively produced by the nucleon nucleon interaction. All microscopic models of the nucleus are based on some models of the basic interaction between two nucleons. Firstly, we construct a potential well by using a phenomenological nucleon-nucleon interaction.

Nucleon-Nucleon Interaction

The starting point to study nuclear many-body theory is two-body nucleon-nucleon

interactions in the nuclear system. We assume that they give rise to an average single- particle potential. The interaction between two nucleons, nucleon 1 and nucleon 2 is expressed by

$$V_{N-N}(r_1 - r_2) = V_0\, f(r_1 - r_2) \qquad (1)$$

Where, V_0 = central depth of the potential

r_1 = radial distance of nucleon 1 from the center of mass

r_2 = radial distance of nucleon 2 from the center of mass and

f describes the shape of the potential.

Let $r = r_1 - r_2$

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$$V_{N-N}(r) = V_0 f(r)$$

The function "f" is assumed to be smooth and has short range. In this calculation, we take "f" to be a Gaussian form,

$$f(r) = e^{-\left(\frac{r}{\beta}\right)^2}$$

where, β = range parameter of the interaction.

Then, the equation becomes, $V_{N-N}(r) = -V_0\, e^{-\left(\frac{r}{\beta}\right)^2}$ (2)

According to the nuclear matter calculation (Ring P. and Shuck P.(1980),), V_0 is obtained to be 50 MeV while β varies from 1 fm to 2 fm.We will find out the value of β which reproduces the experimental binding energy of ^{12}C and then, the potential between a nucleon and the core nucleus ^{11}C will be derived.

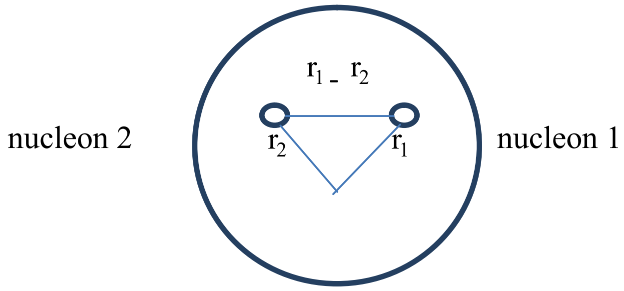

Figure 1.1 Nucleon-Nucleon Interaction

Potential Well of ^{12}C Nucleus

We will derive the nucleon-nucleus potential by folding the nucleon-nucleon interaction with the density distribution of the core nucleus.The average interaction experienced by the nucleon 1 due to the other nucleon is

$$V_{N-Nucleus}(r_1) = \int V_{N-N}(r_1 - r_2)\rho(r_2)d^3r_2$$

$$V_{N-Nucleus}(r_1) = \int V_0 f(r)\rho(r_2)d^3r_2$$

$$r_1 - r_2 = r$$

$$r_2 = r - r_1$$

$$d^3r_2 = d^3r - d^3r_1$$

Therefore, $V_{N-Nucleus}(r_1) = -V_0\ \rho(r)\int e^{-\left(\frac{r}{\beta}\right)^2} d^3r$ (3)

In general, $V_{N-Nucleus}(r) = -V_0\ \rho(r)\int e^{-\left(\frac{r}{\beta}\right)^2} d^3r$

where, $\rho(r)$ = nucleon density distribution.

We will use nucleon density distribution of the nucleus $\rho(r)$ in harmonic oscillator model (Jager C.W., Vires H. & Vires C.(1974)). In the harmonic oscillator model, the density distribution $\rho(r)$ that depends on the radial distance is

$$\rho(r)=\rho_0\left(1+\alpha\left(\frac{r}{a}\right)^2\right)e^{-\left(\frac{r}{a}\right)^2} \tag{4}$$

where, ρ_0 = normalization constant

α = 1.067 fm

a = 1.687 fm.

$\rho(r)$ is normalized as $\int\rho(r)d^3r=z$ where z is the charge number.

$$\int\rho_0\left(1+\alpha\left(\frac{r}{a}\right)^2\right)e^{-\left(\frac{r}{a}\right)^2}d^3r=6$$

Then, we get the normalization constant ρ_0 ,

$$\rho_0=\frac{6}{\left(\pi a^2\right)^{\frac{3}{2}}\left(1+\frac{3}{2}\alpha\right)}$$

By substituting the values of the parameter α and a, we obtain ρ_0 = 0.086. Finally, we obtain nucleon density distribution of the nucleus as follow:

$$\rho(r)=0.086\left(1+\alpha\left(\frac{r}{a}\right)^2\right)e^{-\left(\frac{r}{a}\right)^2}$$

By substituting the above $\rho(r)$ into equation (1), we obtain

$$V_{N-Nucleus}=-V_0\,0.086\left(1+\alpha\left(\frac{r}{a}\right)^2\right)e^{-\left(\frac{r}{a}\right)^2}(\pi)^{\frac{3}{2}}\beta^3 \tag{5}$$

Mathematical Formulation

Schrödinger equation for two-body bound system is

$$\left\{-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}+\frac{\hbar^2}{2\mu}\frac{\ell(\ell+1)}{r^2}+V(r)\right\}u(r)=E\,u(r) \tag{6}$$

To solve the above equation, we expand the wave function u(r) with Gaussian basis. The Gaussian form for wave function is

$$u(r)= r^{\ell+1} \sum_{j=1}^{N_b} c_j \, e^{-\left(\frac{r}{b_j}\right)^2} \tag{7}$$

wherec_j's are expansion coefficients, b_j's are range parameters and N_b is number of basis all of which are to be adjusted in the calculations. b_j's are chosen to be geometric progression as follow,

$$\frac{b_2}{b_1} = \frac{b_3}{b_2} = \frac{b_4}{b_3} = \cdots = c$$

$$b_{j+1} = cb_j$$

$$b_{N_b} = c^{N_b - 1} b_1$$

$$c = \left(\frac{b_{N_b}}{b_1}\right)^{\frac{1}{N_b - 1}}$$

The Schrödinger equation for radial part becomes,

$$\left\{-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2} + \frac{\hbar^2}{2\mu}\frac{\ell(\ell+1)}{r^2} + V(r)\right\} \sum_j c_j \, r^{\ell+1} e^{-\left(\frac{r}{b_j}\right)^2} = E \sum_j c_j \, r^{\ell+1} e^{-\left(\frac{r}{b_j}\right)^2} \tag{8}$$

Multiplying both sides of the equation by $r^{\ell+1} e^{-\left(\frac{r}{b_j}\right)^2}$ from the left and integration through the equation;

$$\int r^{\ell+1} e^{-\left(\frac{r}{bi}\right)^2} \left\{-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2} + \frac{\hbar^2}{2\mu}\frac{\ell(\ell+1)}{r^2} + V(r)\right\} \sum_j c_j \, r^{\ell+1} e^{-\left(\frac{r}{b_j}\right)^2} dr$$

$$= E \int r^{\ell+1} e^{-\left(\frac{r}{bi}\right)^2} \sum_j c_j \, r^{\ell+1} e^{-\left(\frac{r}{b_j}\right)^2} dr \tag{9}$$

$$\sum_j c_j \int r^{\ell+1} e^{-\left(\frac{r}{bi}\right)^2} \left\{-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}\right\} r^{\ell+1} e^{-\left(\frac{r}{b_j}\right)^2} dr + \frac{\hbar^2}{2\mu} \int r^{\ell+1} e^{-\left(\frac{r}{bi}\right)^2} \frac{\ell(\ell+1)}{r^2} r^{\ell+1} e^{-\left(\frac{r}{b_j}\right)^2} dr$$

$$+$$

$$\int r^{\ell+1} e^{-\left(\frac{r}{bi}\right)^2} V(r) r^{\ell+1} e^{-\left(\frac{r}{b_j}\right)^2} dr = E \sum_j c_j \int r^{2(\ell+1)} e^{-\left(\frac{r}{bi}\right)^2} e^{-\left(\frac{r}{b_j}\right)^2} dr \tag{10}$$

In this equation, T_{ij}^{ℓ}, N_{ij}^{ℓ}, and F_{ij}^{ℓ} are the kinetic energy, centrifugal potential energy and potential energy matrix elements which are described by as follow:

$$N_{ij}^{\ell} = \frac{(2\ell+1)!!\sqrt{\pi}}{2^{\ell+2}\left(\frac{1}{b_i^2}+\frac{1}{b_j^2}\right)^{\ell+\frac{3}{2}}} \tag{11}$$

$$F_{ij}^{\ell} = \frac{\hbar^2}{2\mu}\ell(\ell+1)\left(\frac{(2\ell-1)!!\sqrt{\pi}}{2^{\ell+1}\left(\frac{1}{b_i^2}+\frac{1}{b_j^2}\right)^{\ell+\frac{1}{2}}}\right) \tag{12}$$

$$T_{ij}^{\ell} = -\frac{\hbar^2}{2\mu}\left[\begin{array}{l}\frac{4}{b_j^4}\left(\frac{(2\ell+3)!!\sqrt{\pi}}{2^{\ell+3}\left(\frac{1}{b_i^2}+\frac{1}{b_j^2}\right)^{\ell+\frac{5}{2}}}\right)-\frac{4\ell+6}{b_j^2}\left(\frac{(2\ell+1)!!\sqrt{\pi}}{2^{\ell+2}\left(\frac{1}{b_i^2}+\frac{1}{b_j^2}\right)^{\ell+\frac{3}{2}}}\right)+\\ \ell(\ell+1)\left(\frac{(2\ell-1)!!\sqrt{\pi}}{2^{\ell+1}\left(\frac{1}{b_i^2}+\frac{1}{b_j^2}\right)^{\ell+\frac{1}{2}}}\right)\end{array}\right] \tag{13}$$

$$T_{ij}^{\ell} + F_{ij}^{\ell} = \frac{\hbar^2}{2\mu} N_{ij}^{\ell} \frac{(4\ell+6)}{b_i^2+b_j^2} \tag{14}$$

In our calculation we use the folding potential between neutron and core nucleus ^{11}C. Thus the potential energy matrix element with Gaussian basis wave function becomes as follow:

$$V_{ij}^{\ell} = r^{\ell+1} e^{-\left(\frac{r}{b_j}\right)^2} \left[V_{\text{Nucleon-Nucleus}}(r)\right] r^{\ell+1} e^{-\left(\frac{r}{b_j}\right)^2} dr$$

The potential energy matrix element

$$V_{ij}^{\ell} = (-V_0)\rho_0\left(\beta^2\pi\right)^{\frac{3}{2}} \frac{(2\ell+1)!!}{2^{\ell+2}} \frac{\sqrt{\pi}}{\left(\frac{1}{b_i^2}+\frac{1}{b_j^2}+\frac{1}{a^2}\right)^{\ell+\frac{3}{2}}} \times \left[1+\frac{(2\ell+3)}{2}\frac{1}{\left(\frac{1}{b_i^2}+\frac{1}{b_j^2}+\frac{1}{a^2}\right)}\frac{\alpha}{a^2}\right] \tag{15}$$

The Hamiltonian is expressed by summing of kinetic energy term, centrifugal term and potential term as follows:

$$H_{ij}^{\ell} = T_{ij}^{\ell} + F_{ij}^{\ell} + V_{ij}^{\ell} \tag{16}$$

$$\sum_j H_{ij}^{\ell} c_j = E \sum_j N_{ij}^{\ell} c_j \tag{17}$$

By writing equation (15) in terms of matrix form as follows:

$$[H][c] = E[N][c] \tag{18}$$

The above equation (16) is solved iteratively by giving an estimate E_0 value and initial set of $c_i^{(1)}$'s. Therefore, the eigen value equation is obtained.

$$E = E_0 + \frac{C^n}{C^{n+1}} \tag{19}$$

where n+1 is the number of iteration. Then we get the neutron separation energy of ^{12}C.

Result and Discussion

We have formulated the kinetic energy matrix elements, the centrifugal potential matrix elements and the potential matrix elements with Gaussian basis. Then, we solved the two-body Schrodinger equation. The parameters b_1, b_N and N_b are adjusted to get the convergent result. We found that the optimum value of the input parameters are b_1= 0.1, b_N = 20.0, N_b= 40. Then we got the neutron separation energy of ^{12}C.

We have calculated the neutron separation energy of ^{12}C by varying the value of β parameter starting from 1.0 fm and increasing it by 0.1 fm. At β = 1.2 fm, the neutron separation energy is -12.04 MeV and at β = 1.3 fm, it is -18.91 MeV. Therefore we found that the experimental neutron separation energy which is -18.72 MeV (Audi G. and Wapstra A.H.,(1993)) should exist between the β value of 1.2 fm and 1.3 fm. The neutron separation energy at various values of β are shown in table (1).

Table 1 The list of neutron separation energy at various β values

β interaction range parameter (fm)	**Neutron separation energy (MeV)**
1.0	-3.19
1.1	-6.84
1.2	-12.04
1.3	-18.91
1.25	-15.26
1.26	-15.96
1.27	-16.66
1.28	-17.39
1.29	-18.15
1.291	-18.22
1.292	-18.29
1.293	-18.38
1.294	-18.45
1.295	-18.53
1.296	-18.61
1.297	-18.68
1.298	-18.76
1.299	-18.84

At β = 1.29 fm the neutron separation energy is -18.15 MeV which is close to the experimental value. So, the value of β is increased by 0.001 fm between 1.291 fm and 1.299 fm. We found that at β = 1.298 fm, our theoretical neutron separation energy -18.76 MeV is in good agreement with the experimental value (Audi G. and Wapstra A.H.,(1993)).

Conclusion

We have calculated the neutron separation energy of ^{12}C nucleus by solving the two-body Schrodinger equation with Gaussian basis wave function. In our calculation nucleons are assumed to be moving freely in a nuclear potential well. We derived the nucleon-nucleus potential by folding the one range Gaussian nucleon-nucleon interaction with the nucleon density distribution of the ^{12}C nucleus. Then we obtained the neutron separation energy of the last neutron to be -18.76 MeV at V_0 = 50 MeV and β = 1.298 fm in our calculation. We can reproduce the correct neutron separation energy of ^{12}C which is in good agreement with the experimental value -18.72 MeV (Audi G. and Wapstra A.H.,(1993)).

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